



Visualizing radiofrequency electromagnetic field exposure through Voronoi-based maps

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Abstract

Measuring the exposure of radiofrequency electromagnetic fields in a city is a very laborious task. To simplify this process for a city of 200 thousand inhabitants, Voronoi diagrams were used. The city was divided into cells based on the Euclidean distance to a point on the map, and each cell was assigned the value of the electric field measured at that point using a personal exposimeter. The number of cells varied from five to 30 and, finally, one 100. The maps obtained are commented on, using a palette of four uniform colors for the cells, to facilitate their perception. The more cells are considered, the smaller the cells will be. A stabilization of the process is observed as the points within the city map increase. The colors represent the RMS (root mean square) electric field measured at each seed point, extended geometrically to its Voronoi cell for visualization purposes. The colors of the areas remain the same; and in some other areas, cells appear with slightly distinct colors, due to the addition of new measured points. Some cells change color due to these new measurements of the new points. In this study, the predominant color is green, which is the measured field at the seed point was 1.9 V/m RMS (while the maximum allowed by the ICNIRP guidelines is 61.4 V/m). There are three cells in the city with E values above 3.9 V/m, reaching the highest value of 11.4 V/m. The entire city is within the recommended maximum. The Voronoi diagram method is shown to be useful and interesting for the presentation of radiofrequency electromagnetic fields exposure measurements in a medium-sized city. The Voronoi cells do not contain information about the intracell spatial variation of the electric field; each polygon represents the RMS value measured at its generating point (seed), offering a discrete visualization of the spatial distribution of the measurements and illustrating how the electric field levels vary across the study area.

Keywords Radiofrequency electromagnetic fields · Voronoi diagram or Thiessen polygons · Tessellation · Personal exposimeter

Introduction

Understanding and accurately mapping exposure to radiofrequency electromagnetic fields (RF-EMF) is crucial for assessing potential health and environmental impacts. The

increasing deployment of wireless communication infrastructure, particularly in urban environments, has led to a growing interest in methodologies capable of quantifying and visualizing exposure levels. Traditional methods for RF-EMF mapping often rely on interpolation techniques, such as Kriging, which estimate exposure levels based on spatial correlation models (Aerts et al. 2013; Paniagua et al. 2013; Paniagua-Sanchez et al. 2024; Ramirez-Vazquez et al. 2023a, 2023b). However, these methods require significant computational effort and predefined statistical assumptions, which may not always align with real-world spatial distributions.

Voronoi diagrams were introduced by Dirichlet in 1850, but it was not until the twentieth century, in 1907, that the Ukrainian mathematician Georgy Voronoi used them systematically in his work and contributed to their later use in scientific fields (Voronoi 1908, 1907). In 1965, for example, they were used to studying the density of trees in a forest.

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They have also been used to studying the amount of rainfall in an area. In geographic information programs, they are called Thiessen polygons after the work done by the American meteorologist A. H. Thiessen around 1911 studying the amount of rain in certain areas and trying to predict and study the weather (Thiessen 1911). They are currently being used in many different studies and research fields (Mount 2002; Okabe et al. 2000). However, to the best of our knowledge, Voronoi diagrams have not yet been applied in RF-EMF exposure assessment.

If one were to compare Voronoi diagrams (Thiessen polygons) and Kriging, one would identify that there are two techniques used in geostatistical analysis with significant differences in their approach and application. On the one hand, Voronoi diagrams are a geometric technique that uses the deterministic method, which allows dividing a space into regions, each associated with a set known points. Each point in space is assigned to the region of the nearest known point, forming polygons, and its greatest application is in situations where the influence of a point decreases uniformly with distance, such as in geography (distribution of weather stations) and biology (animal territories). Furthermore, Kriging interpolation is a statistical technique that not only considers the values of the known points, but also the structure of spatial variability. It uses a covariance model or semivariogram that describes how the data are spatially correlated, to make the best possible estimate at unsampled points, minimizing the prediction error and providing an estimate of the accuracy. Its application is mostly in mining and hydrology, where an accurate estimate is required, and the spatial variability of the data is known. Based on the information available now, it can be concluded that these two methods are complementary and can be especially useful in the field of RF, just like Kriging interpolation. We are also working on this comparison of methods and intend to have results and publish them in the very near future.

In spatial environmental sciences, Voronoi (Thiessen) tessellations are a recognized exploratory mapping technique when measurements are scarce, irregularly distributed, or when one wishes to avoid introducing artificial spatial correlations that cannot be empirically validated. This approach has been used in meteorology, rainfall distribution, (Thiessen 1911), air pollution monitoring (Rivera-González et al. 2015), and environmental noise exposure (Zhao et al. 2023). Our aim is precisely to apply this transparent, model-free tool to the field of RF-EMF exposure, where assumptions of isotropy and smooth gradients are rarely justified.

The objective of this study is to evaluate the use of Voronoi diagrams in the representation of RF-EMF exposure levels across a medium-sized city. In this first study, the colored polygons represent the RMS (root mean square) electric field measured at each generation point, geometrically extended to its Voronoi cell for visualization purposes. The

study focuses on Albacete, Spain, a city with approximately 200,000 inhabitants and a surface area of about 10 km². By measuring RF-EMF levels at various points and generating Voronoi-based maps, this study aims to identify exposure patterns, validating the approach against in situ measurements. This work also discusses the potential advantages and limitations of Voronoi diagrams in exposure assessment and their applicability in urban RF-EMF studies.

Basic theory

A Voronoi diagram is a partitioning of a plane into regions, called Voronoi cells, based on the proximity to a given set of seed points (known points). Mathematically, consider a set of N points placed on a plane, $p = \{p_1, p_2, \dots, p_N\}$. Any other point q in the plane is closer to one of these points. If Euclidean coordinates and the distance between two points in the XY plane are used, the distance between two points, p_i and q , is defined as follows:

$$|p_i - q| \equiv +\sqrt{(p_{ix} - q_x)^2 + (p_{iy} - q_y)^2} \quad (1)$$

The Voronoi cell $V(p_i)$ of the point p_i is defined as the set of points q of the plane for which the closest of those in the set P is p_i :

$$V(p_i) = \left\{ q : |p_i - q| \leq |p_j - q|, \forall j \neq i \right\} \quad (2)$$

It is said that q is in the zone of influence of p_i when it is closer to p_i than to any other of the N points of P , that is, q belongs to a region of the plane called the Voronoi cell of the point p_i . The set of Voronoi cells is called a Voronoi diagram. The boundary points of the Voronoi cells, obviously, are equidistant from two points. The vertices belong to at least three cells (De Berg et al. 2008; Edelsbrunner 2014).

Each Voronoi cell is associated with a point, such that they cover the entire space without leaving gaps. Voronoi cells cannot overlap; each point in space belongs to one and only one cell, except for boundary points. Voronoi cells tile the area of the plane to be studied. It is a two-dimensional surface, but this tiling can be generalized to higher dimensions. Figure 1 shows a Voronoi diagram corresponding to 13 points in a rectangle.

In this study, Voronoi tessellation is applied to a dataset of RF-EMF measurements taken across the city of Albacete. The city is divided into Voronoi cells, with each cell assigned the electric field intensity measured at its generating point. This method shows a distribution of distances from seed points that vary greatly in field measurement. Seed points are not necessarily local maxima; they were arbitrarily chosen.

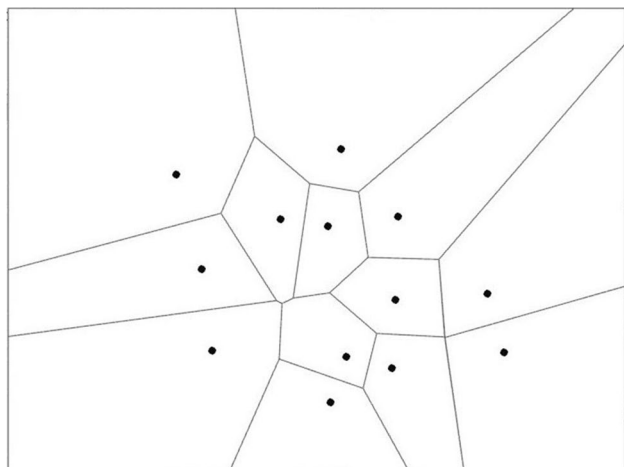


Fig. 1 Example of a 13-point Voronoi diagram in a rectangular plane area. Own elaboration

Our proposal is aimed at applications in public health, environmental epidemiology, and urban planning, which require representative maps of ambient radiofrequency (RF-EMF) exposure obtained from real-world measurements. This approach aims to facilitate the analysis of aggregate exposure in urban areas, which can be useful as an independent variable in health studies or as an environmental indicator in urban planning or regulation contexts.

Materials and methods

Let us suppose a set of n points within the city of Albacete. The electric field (in mV/m) has been measured at each of these points. The Voronoi cell of each of the points is assigned to the value measured at the known point that identifies it, and it will be colored as shown in the generated maps.

Devices and studio microenvironment

The electric field measurements were made with an EME Spy Evolution personal exposimeter, a device that allows measuring up to 20 user-defined frequency bands (Table 1) from among 84 bands predefined by the Microwave Vision Group between 80 MHz and 6 GHz (MVG 2023). This device identifies the contributions of each emitting frequency band and the total field measured. For this case study, the total sum of all frequency bands measured was considered, that is, the total exposure measured by the exposimeter.

EME Spy Evolution is a device that identifies the contribution of each emitter to the total field and is capable of recording measurements over time periods defined by the experimenter. The sensitivity of the exposimeter varies between 0.02 and 0.05 V/m, depending on the frequency band. This device has a tri-axial E-field probe active between 80 MHz and 6 GHz.

Table 1 Twenty frequency bands measured by the EME spy evolution

Band name	Frequency start	Frequency stop
Frequency modulation	87	107
Television network	174	223
Terrestrial trunked radio	380	400
Television network	470	615
LTE (Long-Term Evolution) 700 UL	703	748
LTE (Long-Term Evolution) 700 DL	758	803
LTE (Long-Term Evolution) 800 DL	791	821
LTE (Long-Term Evolution) 800 UL	832	862
GSM (Global System for Mobile Communications)+ UMTS 900 UL	880	915
GSM (Global System for Mobile Communications)+ UMTS 900 DL	925	960
GSM (Global System for Mobile Communications) 1800 UL	1710	1785
GSM (Global System for Mobile Communications) 1800 DL	1805	1880
DECT (Digital Enhanced Cordless Telecommunications)	1880	1900
UMTS (Universal Mobile Telecommunications System) 2100 UL	1920	1980
UMTS (Universal Mobile Telecommunications System) 2100 DL	2110	2170
40 TDD	2345	2355
Wi-Fi 2G	2436.75	2446.75
LTE (Long-Term Evolution) 2600 UL	2493.25	2576.75
LTE (Long-Term Evolution) 2600 DL	2620	2690
Wi-Fi 5G	5150	5850

DL means download and UL means upload

Measurements have been made in the city of Albacete, at different previously selected points, starting with 5 points up to 30 points (Fig. 2a), and to conclude the measurement process, measurements have also been made at 100 points (this appears in the Discussion section for comparison and comment). The selection of the points was random but strategically distributed, trying to cover the entire city. The data obtained has been obtained through spot measurements, using the personal exposimeter, a tripod, and a tablet with the real-time visualization application (Ramirez-Vazquez et al. 2024a). The application allowed us to see the measurements in real-time during the measurement period and record the coordinates to georeference each measured point (Fig. 2b).

The measurement time at each point was 6 continuous minutes without interruption, having configured the exposimeter to make a measurement every 10 s (rms values for 10 s). During this measurement period, the research team that made the measurements carried their mobile phone in airplane mode, to avoid any interference with this process.

Data processing and statistical analysis

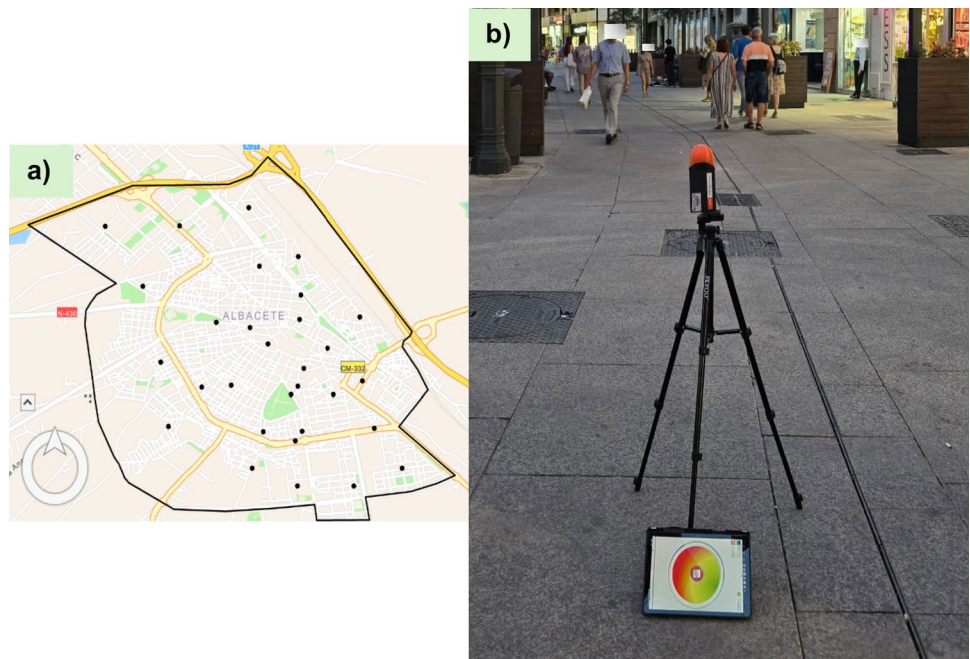
Once the measurement process was completed, the data was downloaded from the real-time application and visualized using the EME SPY Evolution Analysis software and finally exported to a Microsoft Excel worksheet to clean the measurements, verify the coordinates of each measured point, and calculate the average values of said measurements made at each point. Finally, geostatistical analysis and map creation were carried out through the tool Thiessen polygons, also

known as Voronoi cells, using ArcGIS software. The maps presented in the results section were created with measurements taken with the EME-SPY Evolution personal exposimeter, showing the total electric field recorded by all the selected bands (in mV/m). In this first study, the total of the 20 selected bands, frequency bands that operate in Spain, of the 84 frequency bands that the exposimeter is capable of measuring (MVG 2023) has been considered. But the objective is to be able to analyze the bands separately in a future study.

The study was carried out on a normal working day, and the intention was to repeat these measurements on days with fewer people to compare the results. But it is also true that the measurements conducted near the bus and train stations, as will be discussed later, where a very high value was recorded, were repeated at different times of the day and on different days, and as a result, it has been identified that the intensity levels are very similar; there are no significant differences.

Voronoi diagrams or Thiessen polygons, Voronoi tessellation is a transparent and model-free way to divide space when measurements are scarce and one wishes to: (a) communicate spatial heterogeneity without inventing gradients and (b) calculate area- or population-weighted summaries from measurements at chosen points (seeds), the points where the electric field measurement data were collected. Each seed corresponds to a georeferenced location where the time-averaged root mean square (RMS) electric field was recorded using the exposimeter. These points do not represent transmitting antennas or local maxima; they are merely measurement points that define the nearest cell.

Fig. 2 **a** Albacete map and location of the measured points. **b** EME SPY evolution, tripod, and tablet during the measurement process at a specific point on the map, on a pedestrian street in Albacete



Results

Different Voronoi diagrams have been created, varying the number of points and therefore the number of cells. The Voronoi cells do not contain information about the intracell spatial variation of the electric field; each polygon represents the RMS value measured at its generating point (seed), offering a discrete visualization of the spatial distribution of the measurements and illustrating how the electric field levels vary across the study area.

Figure 3 shows the tessellation generated with 5 points, Fig. 4 with 10 points, Fig. 5 with 15 points, Fig. 6 with 20 points, Fig. 8 with 25 points, and Fig. 9 with 30 points. Finally, Fig. 10 will be shown with 100 points (this last map appears in the Discussion section). The central point located within each polygon is the point measurement that has been measured with the personal exposimeter. This is summarized in Table 2.

In Fig. 3, the electric field, E , has been measured at five arbitrary points in the city and, subsequently, the associated Voronoi diagram has been generated. A palette of four uniform colors has been used because this simplifies the interpretation of the average data. Dark green, light green, orange, and blue were selected, in increasing order of electric field value. The blue color is used to highlight the cells

with the highest values, and the dark green color has the lowest values, as indicated in the label of each map. The colors represent the RMS electric field measured at each seed point, extended geometrically to its Voronoi cell for visualization purposes.

In all maps, the colors correspond to the following ranges, according to the values measured and represented in each one: dark green: from 0 to 2000 mV/m; light green: from 2001 to 4000 mV/m; orange: from 4001 to 6000 mV/m; and blue: from 6001 to 12,000 mV/m. Although the first range (dark green) goes from 0 to 2000 mV/m, to highlight the minimum values measured and represented in each map, the minimum measured value has been indicated in the color palette of each map, and as we will see, this minimum value varies in each map.

In Fig. 3, the dark green color is the range of the zone from 305 (lowest value) to 2000 mV/m, and the light green color is the range of the zone from 2001 to 4000 mV/m. The measured values of E range between 305 and 2609 mV/m, whose corresponding intensities are 0.247 and 18.06 mW/m² (Ramirez-Vazquez et al. 2022) values that are quite far from the maximum indicated by the International Commission on Non-Ionizing Radiation Protection (ICNIRP) guidelines (ICNIRP 2020; Ramirez-Vazquez et al. 2024b) which is 10

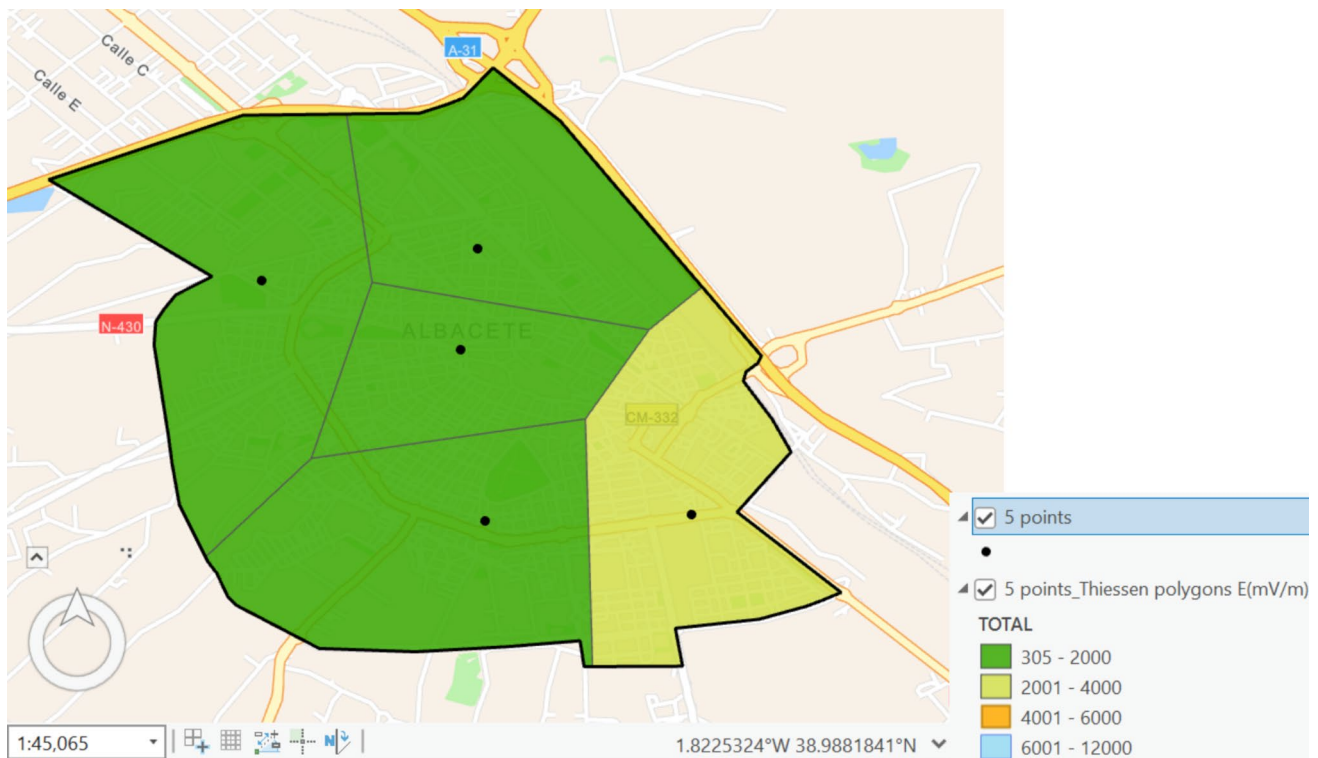


Fig. 3 Map of the city of Albacete with 5 points and their corresponding Voronoi diagram (also known as Thiessen polygons). The distinct colors inform us about the values of the electric field in each zone, in mV/m. In all graphs, we have used a palette of four uniform

colors: dark green, light green, orange, and blue. The blue color is used to highlight the cells with the highest values and the lowest values (dark green), as indicated in the label of each map

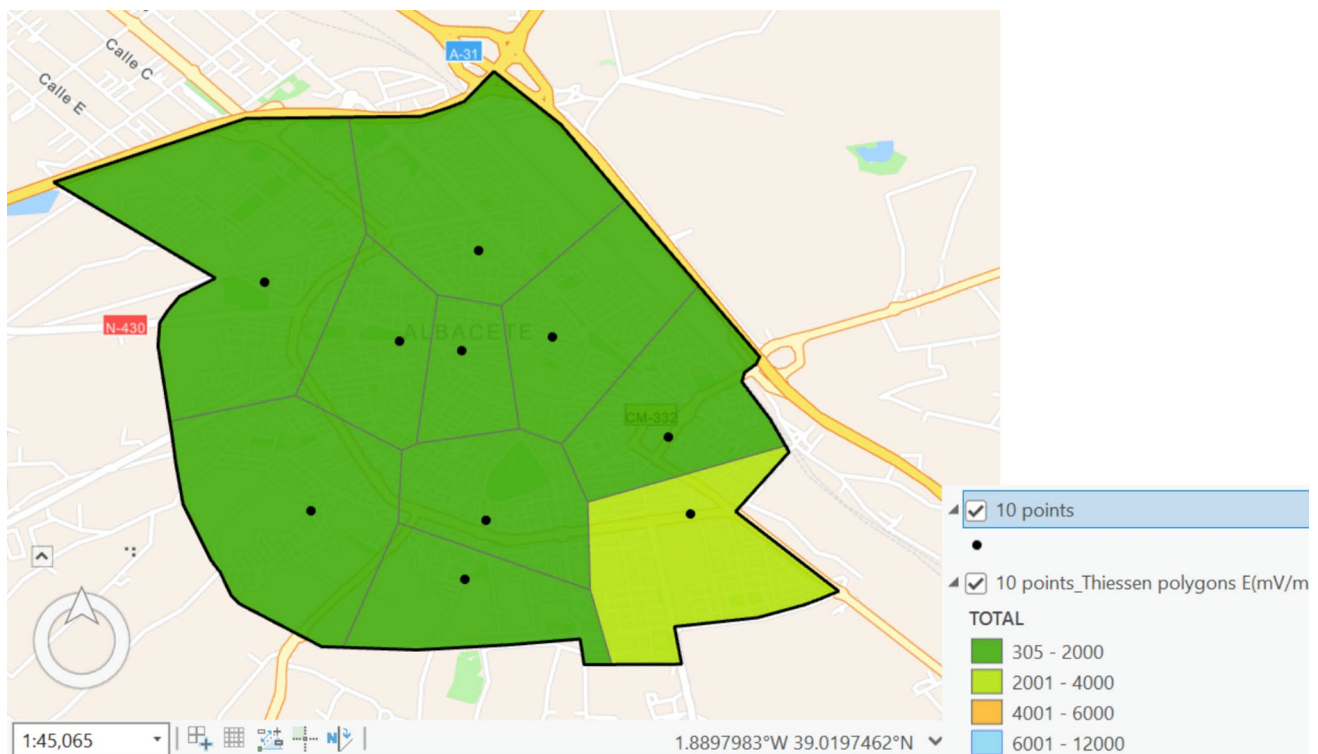


Fig. 4 Map of the city of Albacete with 10 points and its corresponding Voronoi diagram. The color palette informs us about the values of the electric field in each area, in mV/m

W/m^2 . If we adopt a more restrictive point of view, that is, if instead of 10 W/m^2 , we used the value of 2 W/m^2 which corresponds to 27.7 V/m (solid blue line in the attached graph), in no case did the measured values exceed these restrictions (Ramirez-Vazquez et al. 2024b). The highest values are in the southeastern part of the city and the lowest are in the north, and the E values range between 305 and 4000 mV/m (Fig. 3).

The first 5 points have been chosen in such a way that, being arbitrary but strategically distributed, trying to cover the entire city map. The following points were chosen so that little by little more of the map surface is covered. The members of the research group went there to measure, and with the GPS, the latitude and longitude of said point were obtained to then represent it on the city map in a georeferenced way.

Figure 4 shows the same map of the city of Albacete, but now with ten points, the previous five plus five new points. The attempt is to cover more and more areas of the map. The E values range between 305 and 4000 mV/m . The northern area has the lowest E , while the highest values appear again in the southeast. Compared to the previous map, there are areas in the center with the same color (values). Now the light green area ranges from 2001 to 4000 mV/m . When more points are used on the map, more detailed information is obtained, with more areas appearing, which allows a more

accurate study of the map of radiofrequency field intensities, which is really the objective.

Figure 5 a shows the same map of the city of Albacete, but now with 15 points, the previous ten plus five new ones. The dark green zone (whose values are 217–2000) has grown from the north. Previously, the green zone covered up to 305 mV/m in the case of 10 points and up to 305 mV/m in the case of 5, so there are more Voronoi cells that have been colored green. The values of the small squares that appear to the right of the figures are not static; they vary with the number of points in which they have been measured.

To the southeast, a point has appeared with a value in the light green range, so its zone is green as it was in Fig. 4. As the number of points increases, more details appear on the maps. Therefore, the greater the number of Voronoi cells, the more information is obtained. This information is provided by the Voronoi diagram, but you should pay attention to these color changes because they mean that you are looking at more detail and can see more structures that were previously unnoticed.

In Fig. 5a and b, a blue region appears spanning the range from 6001 to $12,000 \text{ mV/m}$. The blue zone is in the south part due to measurements near a large shopping center with many people.

In addition to Fig. 5a, b has been added, in which the same physical information can be observed. The difference

5a

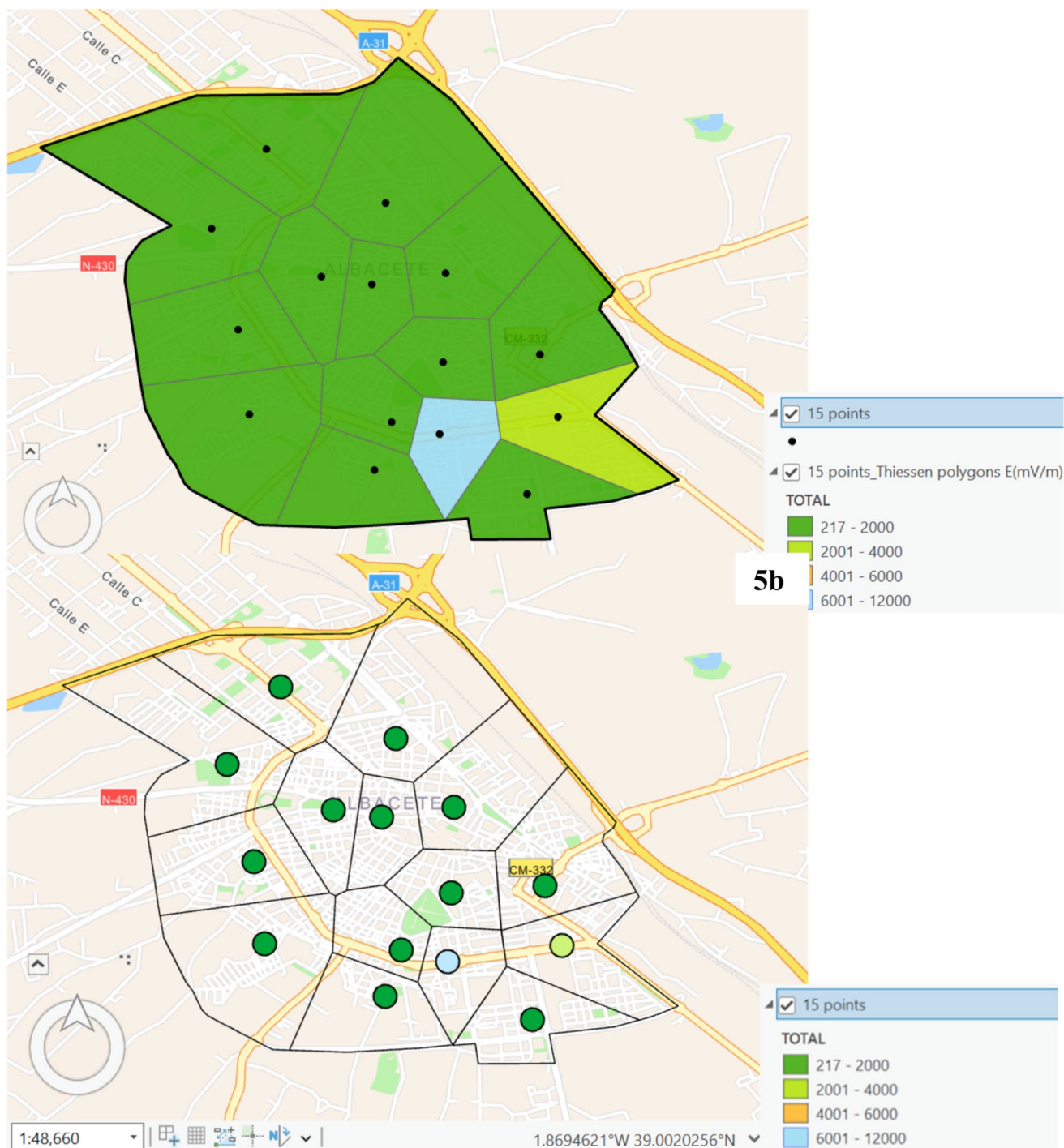


Fig. 5 **a** Map of the city of Albacete with 15 points and its corresponding Voronoi diagram. The color palette informs us about the values of the electric field in each Voronoi cell, in mV/m. **b** Same map of the city of Albacete with the corresponding location of the

seeds that generate the 15 Voronoi cells, using the same color and scale. The color palette informs us of the electric field values measured at each seed point, in mV/m

is that in the first, all the cells are colored, and in the second, the seeds are colored. The purpose of the Voronoi version is to provide an intuitive visualization of spatial coverage

and to delineate areas associated with each measurement for potential integration with demographic or urban layers (e.g., population density, land use). We believe this comparison

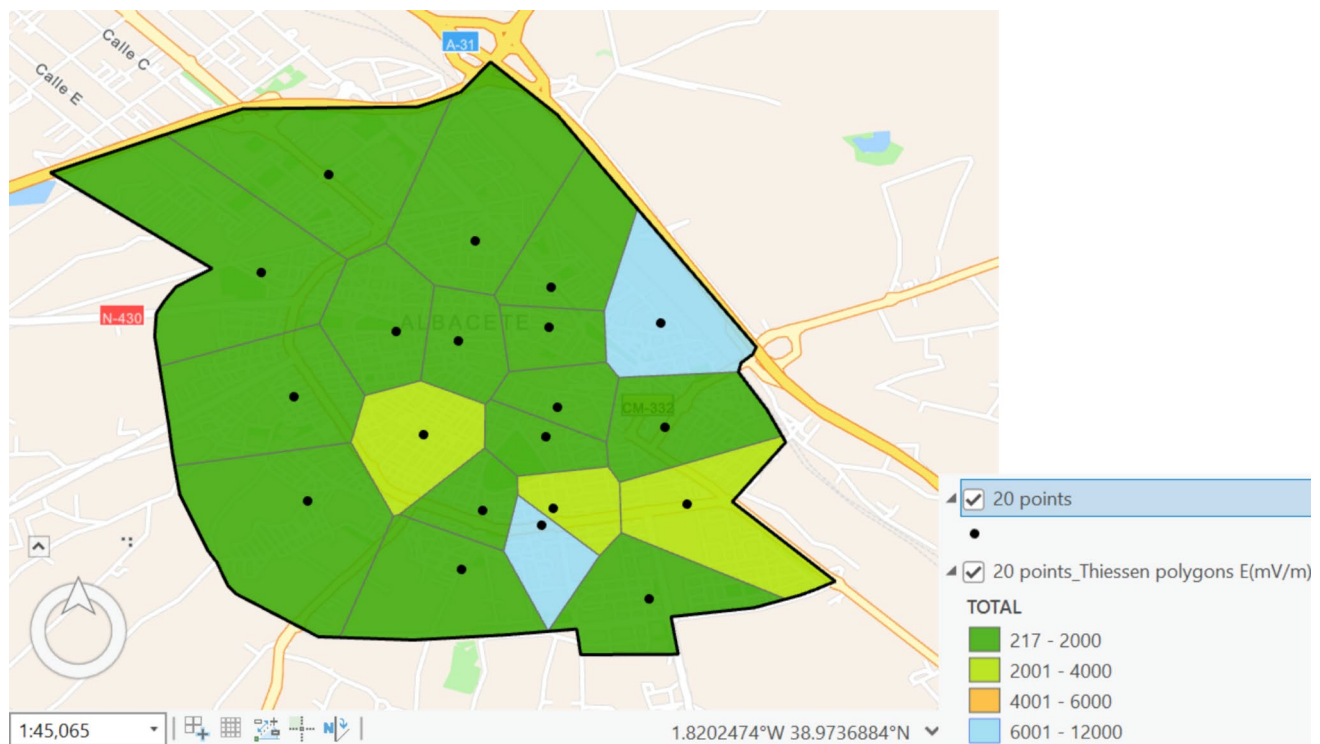


Fig. 6 Map of the city of Albacete with 20 points and its corresponding Voronoi diagram. The color palette informs us about the values of the electric field in each area, in mV/m

Table 2 Figures that have been generated and the Voronoi cells they contain, which match the number of points measured

Figure number	Voronoi's cells
3	5
4	10
5	15
6	20
8	25
9	30
10	100

strengthens the transparency and interpretability of the method.

Figure 6 shows the same map of the city of Albacete, but now with 20 points, the previous 15 plus five new ones. Green is still the predominant color, but there is a blue area to the east of the city with the highest value measured so far, range 6001 to 12,000 mV/m, near the railway line, which borders the city to the east. The exact value was 11,390 mV/m are equivalent to 0.344 W/m^2 (Ramirez-Vazquez et al. 2022), still far from the maximum allowed by ICNIRP, which is 10 W/m^2 (ICNIRP 2020). In that area, there is a large concentration of antennas, shown in Fig. 7, where the mast supports the antennas of different

mobile telephone operators. In Fig. 7, the exposimeter is briefly shown resting on a surface in that photograph, but only for the purpose of capturing the illustrative image near a telephone base station, not during the measurement. It is highlighted that all measurements were taken with the exposure meter mounted on a non-metallic, wooden tripod, as described and shown in Fig. 2b.

The uplink and downlink traffic of this antenna is responsible for this measured value. The maximum value allowed by ICNIRP for the intensity of an electromagnetic wave is 10 W/m^2 , which translates into an electric field value of 61.4 V/m, so 11.39 V/m is 19% of this maximum (Ramirez-Vazquez et al. 2024b). This is the highest value measured, but it is within the allowed range.

In Fig. 8, the same map of the city of Albacete appears, but now with 25 points, the previous 20 plus five new ones. Green continues to be the predominant color, and the two blue zones remain; the map is very similar to the previous map (Fig. 6). It is observed how the structure of the map in the Voronoi cells is acquiring stability.

Figure 9 shows the same map of the city of Albacete, but now with 30 points, the previous 25 plus five new ones. Green remains the predominant color, and the two blue zones remain again, similar as map of Figs. 6 and 8.



Fig. 7 Telecommunications antenna in the blue zone of Figs. 5, 6, 8, 9, and 10. The mast supports the antennas of different mobile telephone operators. In the foreground, the exposimeter is shown in the measurement process

Discussion

In this study, maps with 5, 10, 15, 20, 25, and 30 points have been produced, and a tendency towards stabilization has been identified in the figures generated. It is highlighted that in all graphs, we have used a palette of four uniform colors: dark green, light green, orange, and blue. The blue color is used to highlight the cells with the highest values and the dark green lowest values, as indicated in the label of each map. The colors represent the RMS electric field measured at each seed point, extended geometrically to its Voronoi cell for visualization purposes.

In relation to the measurement process, each measurement was taken over a period of 6 min per point (according to ICNIRP), with one reading every 10 s, providing a set of 36 independent data points, allowing the calculation of a representative mean value of the exposure at each location. Furthermore, this number of samples allows the estimation of the statistical uncertainty associated with the average, using the expression $u = \sigma/\sqrt{n}$, where σ is the standard deviation of the 36 measurements and $n = 36$. This procedure contributes to reducing the overall uncertainty of the

measurements and to better characterizing the local variability of the electric field in urban environments. If we compare this statistical uncertainty with the mean values, we obtain values between 2 and 5%.

To evaluate this stability, a map is generated with 100 points distributed throughout the city (Fig. 10), where the green color predominates. Three cells appear in blue, covering the range from 6001 to 12,000 mV/m, the highest interval, and one of them in orange, with a value between 4001 and 6000 mV/m. These values have been measured and shown in this new map, close to the values shown in the map in Fig. 6 (near the antenna shown in Fig. 7) and values obtained near a shopping center mentioned in Fig. 5. In addition, in this new map, a new high area (blue) has also appeared to the south, near the university area, where there is an antenna on the terrace of a hotel that provides telephone service to the University of Castilla-La Mancha and other nearby leisure establishments.

Figure 11 shows a violin diagram that collects all the data obtained, the 100 points measured, to compare the different maps. A box diagram has been superimposed inside the violin diagram in which the outliers can be seen. The probability density of the data remains practically constant with respect to the median value, but when a value with a high electric field appears, all the violins stretch to include it in their whiskers. This confirms the hypothesis that as the number of points increases, the maps converge towards stable values. The violin diagram shows the mean value and the median of the data; the value that appears closest to the x -axis is the median, and the one that appears above it is the mean. In Table 3, we can see the descriptive values of the measurements taken and presented on each point map, E (mV/m) and I (mW/m²), including the dominant frequency band at each measurement site, to compare them with the reference limits allowed by the ICNIRP.

In relation to the previous map (Fig. 10), it is also important to highlight that in the northern area, there is a lower concentration of both people and commercial establishments. In fact, in the southern area, where network values are observed, there is a shopping center, a football stadium, and a university, a place where it is known that there is already 5G coverage.

Voronoi diagrams can be extremely useful in RF studies, especially for analyzing measured exposure levels, for example, to know, identify, and analyze the distribution of base stations, since, in the field of telecommunications, Voronoi diagrams help to determine the coverage areas of different base stations. The coverage area of a BS is determined by the envelope of usable signal strength, which is not shown in the diagram.

Furthermore, for interference analysis, Voronoi diagrams can be of significant help in identifying areas where interference between different base stations may be a problem, and

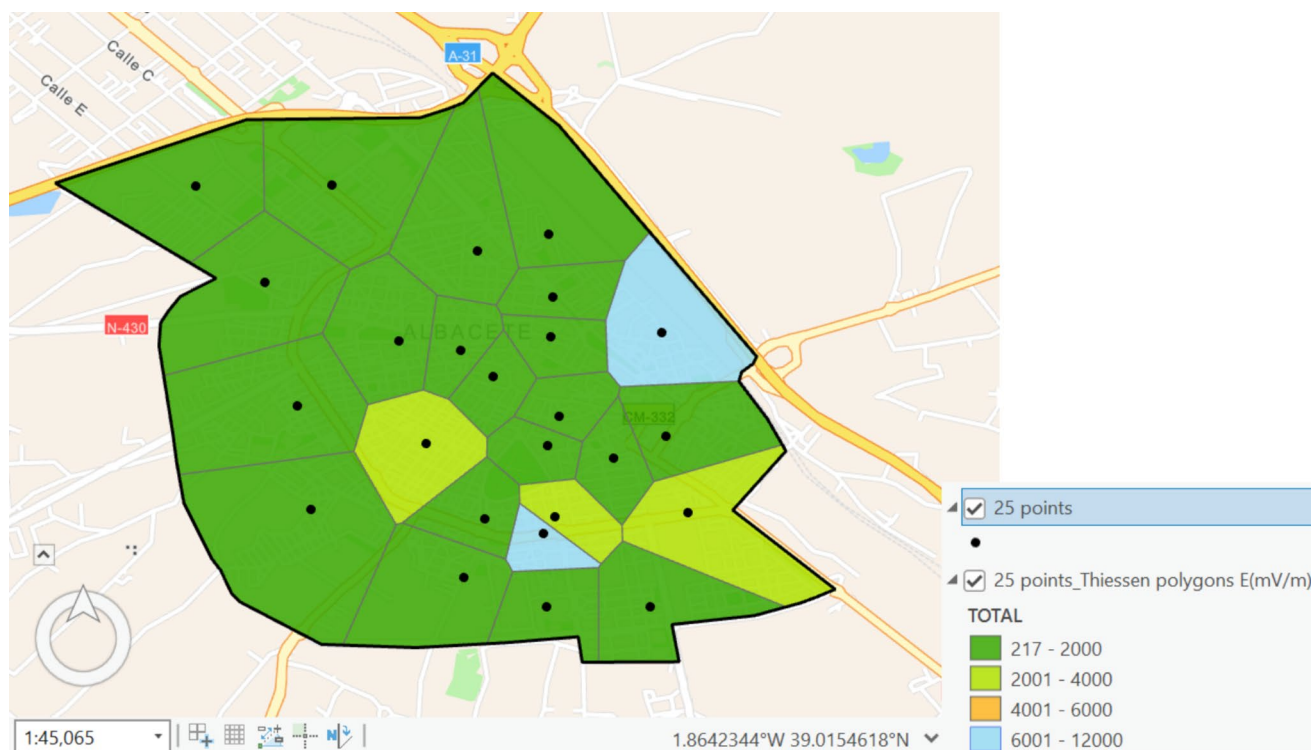


Fig. 8 Map of the city of Albacete with 25 points and its corresponding Voronoi diagram. The color palette informs us about the values of the electric field in each area, in mV/m

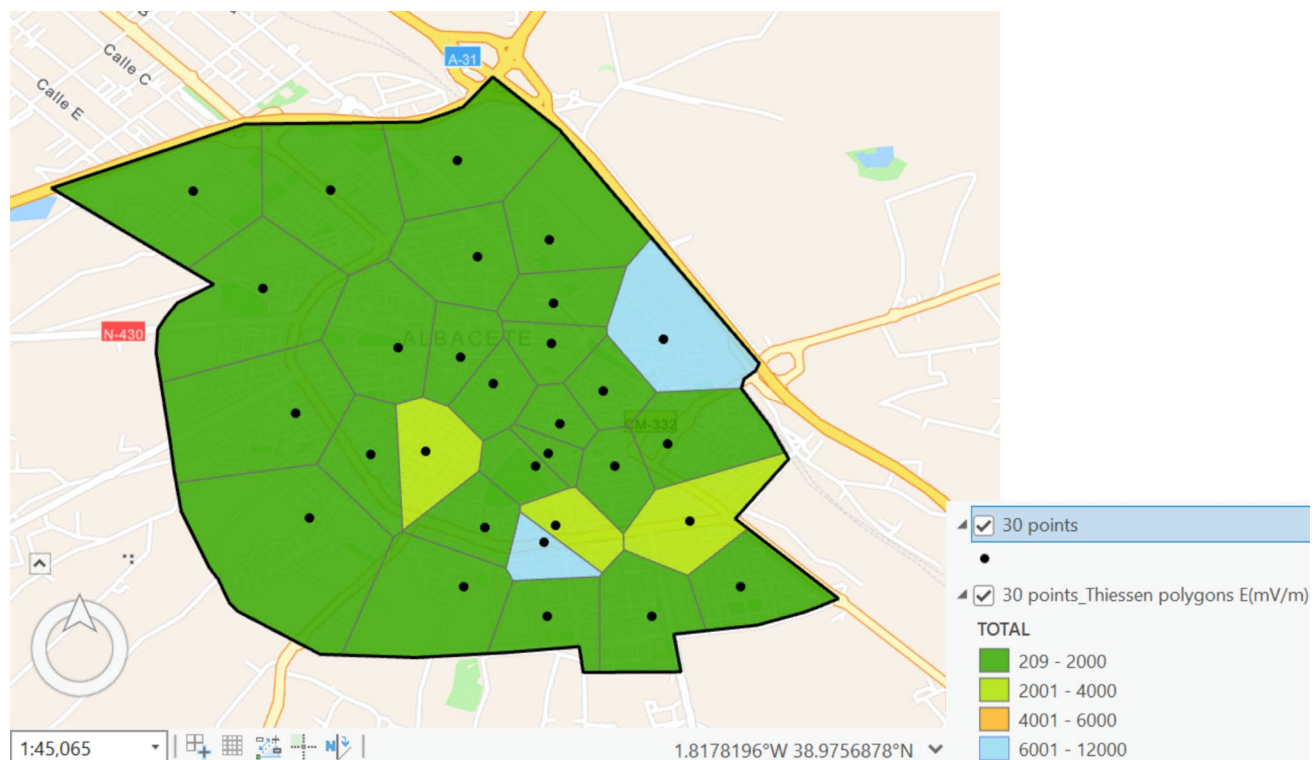


Fig. 9 Map of the city of Albacete with 30 points and its corresponding Voronoi diagram. The color palette informs us about the values of the electric field in each area, in mV/m

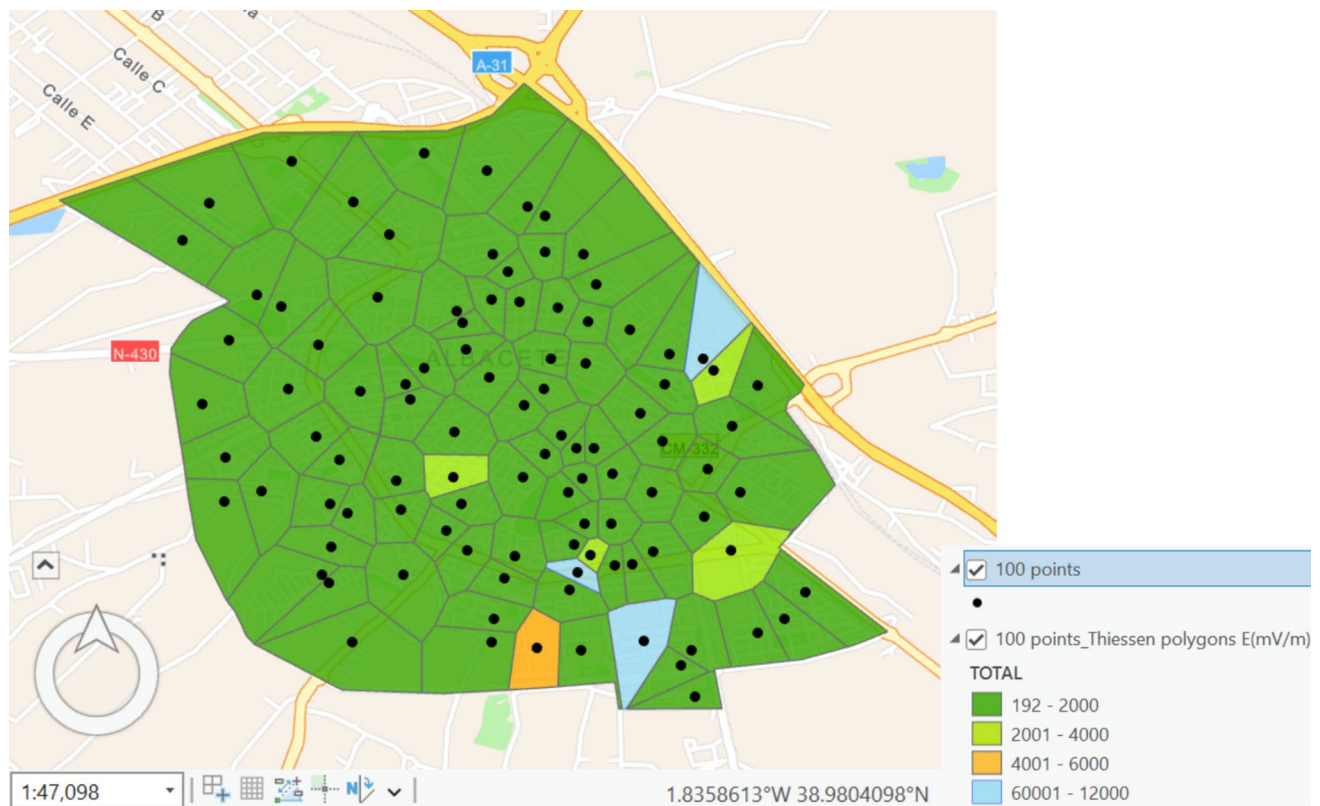


Fig. 10 Map of the city of Albacete with 100 points and its corresponding Voronoi diagram. The color palette informs us about the values of the electric field in each area, in mV/m

in this case, it would be visualized with the tessellated polygons with the intensity levels. By visualizing the regions of influence, informed decisions can be made about the location and configuration of base stations to minimize interference.

The Voronoi diagram-based methodology is fully feasible for replication in other cities, especially in medium-sized or small urban areas, including in large cities, where the number of measurement points may be limited and the need for efficient and rapid coverage is critical. Its main competitive advantage compared to traditional methods like Kriging lies in its simplicity and lower computational cost, as it does not rely on complex statistical models or prior assumptions about spatial correlation structures. This makes it a straightforward and practical tool, particularly suitable for exploratory studies or quick monitoring campaigns in urban environments. Additionally, by providing a direct and geometrically intuitive partition of space, it facilitates the visual interpretation of exposure patterns, making the results more accessible for local authorities and public health decision-makers, enhancing its practical applicability.

In addition, Voronoi diagrams can also be used to optimize the placement of sensors or exposure monitors. This ensures that each sensor covers a specific area without unnecessary overlapping, improving monitoring efficiency.

This can be useful for assessing exposure levels in different areas and taking preventative measures. To the best of our knowledge, and after having performed a search for other research related to RF-EMF and Voronoi or Thiesen polygons, Voronoi diagrams have not been used in studies related to radiofrequency.

This work can offer a representation of ambient RF exposure levels in a city, useful for epidemiological studies, as a spatial exposure variable; longitudinal environmental monitoring; integration into geographic information systems for urban planning; or studies of multiple exposures (noise, pollution, electromagnetic fields, etc.).

We believe that spatial representation based on Voronoi diagrams, constructed from real exposure data, can be useful in preliminary environmental diagnostic studies, as well as in territorial assessment processes or decision-making in urban contexts where detailed technical models are unavailable. As can be seen, Figs. 5a and b contain identical physical information, with Fig. 5a standing out and showing the purpose of the Voronoi version, which is to provide an intuitive visualization of the spatial coverage and delimit the areas associated with each measurement for possible integration with demographic or urban layers (e.g., population density, land use).

Fig. 11 Violin diagram of the electric field data, in mV/m, measured in the city of Albacete. A box diagram has been superimposed inside each violin to show the outliers

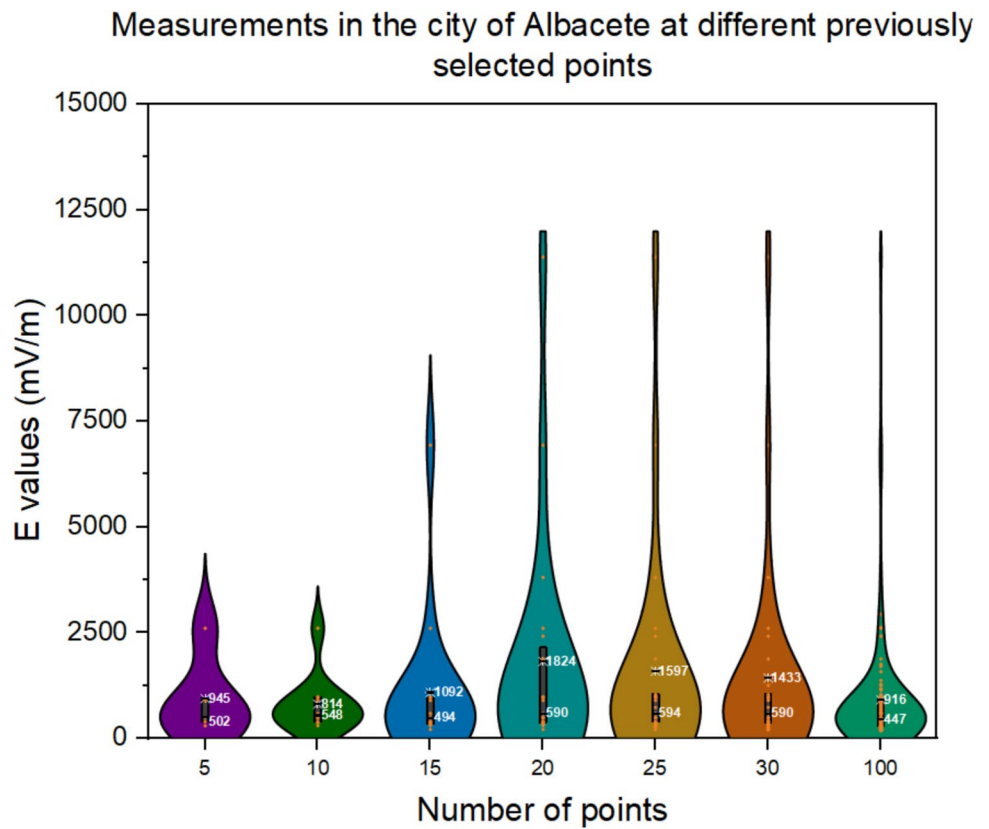


Table 3 Descriptive values of the measurements taken and presented on each point map, E (mV/m), I (mW/m²), and dominant frequency band

Measurement points on the map	Mean		Median		Minimum		Maximum		Dominant frequency band
	E (mV/m)	I (mW/m ²)	E (mV/m)	I (mW/m ²)	E (mV/m)	I (mW/m ²)	E (mV/m)	I (mW/m ²)	
5	945	2.37	502	0.67	305	0.25	2609	18.1	TETRA
10	814	1.76	548	0.80	305	0.25	2609	18.1	UMTS 2100 DL
15	1092	3.16	494	0.65	217	0.12	6940	128	GSM 1800 DL
20	1824	8.82	590	0.92	217	0.12	11,390	344	GSM 900 DL
25	1597	6.76	594	0.94	217	0.12	11,390	344	GSM 900 DL
30	1433	5.45	590	0.92	209	0.12	11,390	344	GSM 900 DL
100	916	2.23	447	0.53	192	0.10	11,390	344	GSM 900 DL

TETRA terrestrial trunked radio, used by security and emergency forces, *UMTS* Universal Mobile Telecommunications System 2100 DL, used for 3G and 4G, *GSM* Global System for Mobile Communications 1800 DL, *GSM* Global System for Mobile Communications + UMTS 900 DL. DL means download

Conclusion

The aim was to carry out additional measures to obtain a detailed map of RF-EMF exposure throughout the city of

Albacete, which has about 200,000 inhabitants and an area of about 10 km². In this work, the Voronoi cell method was applied to generate exposure maps. Several points have been selected on the city map, varying the number of points to see how they change: 5, 10, 15, 20, 25, 30

and, finally, 100 points. Voronoi cells have been obtained for each point, which decrease in size as the number of points increases. The electric field at the point considered has been measured, and this value is assigned to the entire Voronoi cell. The cell has then been colored with one of the four colors considered, according to the value of the measured electric field.

Some cells are colored light green, with intermediate values of the electric field. The cells in blue correspond to the highest average values in the city. The measured electric field is within the range 192–11,390 mV/m. The highest value is almost 60 times the lowest value, which means that there is good variability within the city. If we adopt a more restrictive point of view, that is, if instead of 10 W/m², we used the value of 2 W/m² which corresponds to 27.7 V/m, in no case did the measured values exceed the reference levels established by ICNIRP. The generated maps clearly show a predominance of green, both dark and light green, so it can be concluded that the exposure of Albacete citizens to RF-EMF is well below the maximum recommended by the ICNIRP.

As a result of the study, representative maps of radiofrequency electromagnetic field (RF-EMF) exposure levels were generated using Voronoi diagrams. While the method does not interpolate between measurements, it provides a model-free spatial representation that avoids introducing statistical artifacts and can be a first-level mapping tool preceding advanced interpolation or Kriging analyses.

By progressively increasing the number of sampling points, Voronoi cells were obtained for each point, the size of which is inversely proportional to the number of measured points. This allows for greater stability of the colors indicating RF-EMF exposure levels on the maps, which facilitated the identification of exposure patterns in the study area. This tool offers an empirical and visually intuitive representation of ambient RF exposure levels in a city, which can be used for epidemiological studies, as a spatial exposure variable; longitudinal environmental monitoring; and integration into geographic information systems for urban planning or studies of multiple exposures (noise, pollution, electromagnetic fields, etc.).

As future work and as follow-up to this one, we intend to measure many more points in the city to obtain an increasingly precise exposure map of the entire city of Albacete. In addition to increasing the spatial density of measurements, it also seems truly relevant to analyzing the frequency bands separately, identifying which technologies (e.g., 4G, 5G, broadcast services) contribute the most total exposure. This band-specific analysis would provide a more granular understanding of the exposure landscape, which is particularly important in the context of 5G deployment, where the introduction of millimeter waves and beamforming technologies may lead to more heterogeneous exposure patterns. Furthermore, a comparative analysis between the Voronoi and Kriging methods is also planned.

Author contribution All the authors, E. A., R. R-V., and I. E., made a substantial contribution to this manuscript.

E. A., R. R-V., I. E., have collaborated in the drafting of the manuscript and have discussed the results and the implications of the manuscript at all stages. All the authors have read the manuscript critically and approved it for final submission.

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Data availability All relevant data and material are presented in the main paper.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

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